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Spectroscopy of lower spin states in tellurium isotopes with the inclusion of hexadecapole interaction

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Abstract : Variation After Projection (VAP) calculations in conjunction with Hartree Bogoliubov (HB) ansatz have been carried out for $^{110-116}$Te isotopes. In this framework, the Yrast spectra, B ($E2$) transition probabilities, quadrupole deformation parameter (β_2) and occupation numbers for various shell model orbits have been obtained. The results of the calculation give an indication that it is important to include the hexadecapole-hexadecapole component of the two-body interaction for obtaining various nuclear structure quantities in these tellurium isotopes.

Keywords : Nuclear structure $^{110-116}$Te, variation-after-projection (VAP) calculations, B ($E2$) transition probabilities, quadrupole deformation parameter (β_2).

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1. Introduction

In the past decade, there has been a large interest in the study of ground and excited state properties of even-even tellurium isotopes. The neutron-proton interaction is known to play a dominant role in quadrupole correlations in nuclei. As a result of this, the excitation energies of collective quadrupole excitations in nuclei near a closed shell are strongly dependent on the number of nucleons outside the closed shell. The even mass tellurium (Te) isotopes have only two protons outside the closed Z = 50 shell. The interpretation in terms of a collective model is given in Refs. [1–5]. The even tellurium isotopes with Z = 52 show vibrational-like characteristic at low spin [6] and there is no clear evidence for collective rotational structures in these tellurium isotopes.

Some theoretical models [7–11] have been proposed to explain the energy spectra and the electromagnetic properties of these nuclei. These model calculations cannot be regarded as ultimate calculations as they suffer from some limitations *e.g.* the observed static quadrupole moments [1] of the 2_1^+ state have not been predicted by simple vibrational model and a more complicated theoretical approach by Sorensen [7] fails to predict the negative values observed for this quantity. Sometime back, the surface δ-interaction model was tested for doubly open shell nuclei [9] and found to give satisfactory predictions of B ($E2$; $0^+ \rightarrow 2^+$) values for $^{122-130}$Te, however, they did not make predictions for $^{110-116}$Te. Sambataro [12] attempted to reproduce the energy spectra and electromagnetic transition properties of doubly even tellurium nuclei in the framework of Interacting Boson Approximation (IBA-2). This model gives correct predictions for the collective states but the agreement is very poor for the non-collective states. Apart from IBM-study, Sau *et al* [10] have tried to invoke shell model for studying the properties of the tellurium nuclei. It is noted that this model is practically successful in explaining the low-lying spectra.

Sometime back, the study of tellurium isotopes led to the discovery of new neutron deficient isotopes such as ^{108,110}Te. The systematics of the ground state bands of the even $^{108-124}$Te isotopes (N = 56–72) are given by Paul and coworkers [13–15]. The $E(4^+)/E(2^+)$ energy ratios are found to fall in the range of 2.00 and 3.33. The

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experimental values all lie close to the vibrational limit, *e.g.* the $E(4^+)/E(2^+)$ energy ratio in ^{112}Te is 2.13 which is much closer to the pure vibrational limit (2.00) than to the rotational limit (3.33).

From the excitation energy systematics, it is observed that the position of the first 2^+ state in $^{110-116}$Te remains nearly constant. It is also very interesting to note that same behavior has been adopted by 4^+ state. The energy ratio $E(4^+)/E(2^+)$ are found remarkably constant ($1.945 \leq E(4^+)/E(2^+) \leq 2.095$) for the doubly even tellurium nuclei with mass number A = 110–116. Thus, these nuclei can be categorized as vibrational type of nuclei. In case of energy systematics of 6^+ state, it is observed that they decrease in steps for the nuclei with mass number A = 110–116.

Thus, from the theoretical overview of the work in this region, it is very clear that this region has given an excellent opportunity for testing the validity of various nuclear models and suitability of two-body interactions. In this regard, it was shown by Rani and Khosa [16] that the VAP technique and quadrupole-quadrupole-plus-pairing model of interaction (VQP) are fairly reliable for the calculation of various nuclear structure quantities. The energy gap ($E_2^+ - E_0^+$) is seen to be reproduced with acceptable discrepancy. In the case of Yrast 4^+ state, their absolute values are not reproduced. The agreement for higher spins is also bad. This shows that there is a need for improving upon quadrupole-quadrupole-plus-pairing model of interaction so that a reasonably good agreement with the experiments can be achieved. This has therefore, been a motivation for carrying out the present piece of work.

The purpose of the present work is to know whether quadrupole-quadrupole-plus-pairing model of interaction can further be modified to produce better results in agreement with experiments. Besides this, more useful data on the energy values of higher states is also now available *e.g.*, the energy spectra in ^{116}Te is at present known up to 16^+ [17].

We have examined the available Yrast spectra in neutron deficient Te isotopes with A = 110–116 in the framework of Variation-After-Projection (VAP) technique in conjunction with the HB ansatz for the trial wave functions resulting from the pairing-plus-quadrupole-quadrupole-plus-octupole-octupole-plus-hexadecapole-hexadecapole interaction (VQPOH). The VAP prescription selects an optimum intrinsic state for each Yrast level through a minimization of the expectation value of the Hamiltonian with respect to the states characterized by a definite angular momentum.

Our VAP calculations performed with VQPOH model of two-body interaction show a marked improvement in agreement with the experimentally observed Yrast spectra as compared to the Yrast spectra obtained with the VQP interaction. The results obtained for B ($E2$) transition probabilities and quadrupole deformation (β_2) are also found to be in reasonably good agreement with the available data.

2. Calculational details

The one- and two-body parts of Hamiltonian :

The spherical single particle energies (S.P.E.'s) that we have taken are (in MeV) same as Ref. [18] *i.e.* $(2d_{5/2}) = 0.0$, $(3s_{1/2}) = 1.4$, $(2d_{3/2}) = 2.0$, $(1g_{7/2}) = 2.5$, $(1h_{11/2}) = 4.0$, $(2f_{7/2}) = 10.9$, $(1h_{9/2}) = 11.5$, $(1i_{13/2}) = 13.5$. The S.P.E.'s of $2d_{5/2}$, $3s_{1/2}$, $2d_{3/2}$, $1g_{7/2}$ and $1h_{11/2}$ are exactly the same as that employed by Vergados and Kuo [19] as well as Federman and Pittel [20]. The S.P.E.'s of $2f_{7/2}$, $1h_{9/2}$ and $1i_{13/2}$ orbits are taken from Nilsson diagrams, published in the book [21] with small variations.

The two-body effective interaction that has been employed is of pairing-plus-quadrupole-quadrupole-plus-octupole-octupole-plus-hexadecapole-hexadecapole (VQPOH) type. The parameters of pairing-plus-quadrupole-quadrupole (VQP) part of the two-body interaction are

$$\chi_{nn} \; (= \chi_{pp}) = -0.0139 \text{ MeV } b^{-4} \text{ and}$$
$$\chi_{np} = -0.0262 \text{ MeV } b^{-4} .$$

The relative magnitudes of the parameters of the hexadecapole-hexadecapole parts of the two body interaction were calculated from a relation suggested by Bohr and Mottelson [22]. According to them, the approximate magnitude of these coupling constants for isospin T = 0 is given by

$$\chi_\lambda = [4\pi/(2\lambda + 1)] \times [m\omega_0^2/(A\langle r^{2\lambda-2}\rangle)], \text{ for } \lambda = 1, 2, 3, 4 \quad (1)$$

and the parameters for the T = 1 case are approximately half the magnitude of their T = 0 counterparts. The values of these parameters for octupole-octupole interaction and hexadecapole-hexadecapole interaction are the same as that employed by us in barium isotopes [18].

The procedure for obtaining the axially symmetric Hartree Bogoliubov (HB) intrinsic states has been discussed in Ref. [23]. The methods developed by Onishi and Yoshida [24] for carrying out projection of the states

of good angular momentum from axially symmetric HB intrinsic states has been used for obtaining the spectra.

The VAP calculations have been carried out as follows. We first generated the self-consistent HB solutions $\Phi(\beta)$, by carrying out the HB calculations with the Hamiltonian $(H-\beta Q_0^2)$, where 'β' is a parameter. The selection of the optimum intrinsic states $\Phi_{opt}(\beta_J)$, is then made by finding out the minimum of the projected energy

$$E_J(\beta) = <\Phi_0(\beta)|HP^J_{00}|\Phi_0(\beta)>/<\Phi_0(\beta)|P^J_{00}|\Phi_0(\beta)> \quad (2)$$

as a function of β. In other words, the optimum intrinsic state for each Yrast J satisfies the condition.

$$\partial/\partial\beta|[<\Phi_0(\beta)|HP^J_{00}|\Phi_0(\beta)>/ <\Phi_0(\beta)|P^J_{00}|\Phi_0(\beta)>]|_{\beta=\beta J} = 0. \quad (3)$$

3. Deformation systematics in tellurium isotopes

The mechanism of the observed deformation systematics in tellurium region is examined in the framework of Hertree-Bogoliubov (HB) method employing a valence space which is large enough to permit a systematic and unbiased study of $^{110-116}$Te nuclei. The nucleus ^{100}Sn (Z = 50, N = 50) has been considered as inert core. The results of HB calculations and intrinsic quadrupole moments in $^{110-116}$Te isotopes are presented in Table 1.

Table 1. The experimental values of excitation energy of the E_2^+ state (ΔE) in MeV and intrinsic quadrupole moments of the HB states in $^{110-116}$Te isotopes with the VQP and VQPOH interactions. Here $<Q_0^2>_\pi$ ($<Q_0^2>_\nu$) gives the contribution of the protons (neutrons) to the total intrinsic quadrupole moment. The quadrupole moments have been computed in units of b^2, where $b = \sqrt{\hbar/m\omega}$ is the oscillator parameter.

Te nuclei (A)	E_2^+ *(Exp.)	Interaction: VQP $<Q_0^2>_{HB}$	VQP $<Q_0^2>_\pi$	VQP $<Q_0^2>_\nu$	VQPOH $<Q_0^2>_{HB}$	VQPOH $<Q_0^2>_\nu$	VQPOH $<Q_0^2>_\nu$
110	0.65	39.34	13.94	25.40	63.67	14.39	49.28
112	0.68	42.67	14.07	28.60	63.98	14.46	49.52
114	0.70	49.58	14.23	35.35	64.05	14.45	49.60
116	0.67	58.47	14.38	44.09	64.20	14.47	49.73

*Experimental data taken from Ref. [27].

Discussing first the results for $<Q_0^2>$ moments of the intrinsic states associated with the ground states, one observes that $<Q_0^2>$ with VQP interaction goes on increasing from A = 110 to 116 tellurium isotopes whereas with VQPOH interaction, $<Q_0^2>$ values show a very slow change from A = 110 to 116. Thus, the relative magnitudes of the intrinsic quadrupole moments for Te-nuclei with VQPOH interaction indicate that these nuclei have almost the same amount of deformation and fall in the vibrational region.

Regarding the occupation probabilities of tellurium isotopes, our calculations (see Tables 2 and 3) reveal that the observed behaviour of 2^+ state in $^{110-116}$Te isotopes is intimately connected with the way the occupation probabilities of $(1h_{11/2})_\nu$ and $(2d_{5/2})_\nu$ orbits in the underlying valence space change. It is noted that with the VQPOH interaction, the occupation probability of $(1h_{11/2})_\nu$ orbit increases from 1.47 in ^{110}Te to 3.07 in ^{116}Te, whereas that of $(2d_{5/2})_\nu$ orbit increases from 3.11 to 4.78. The increase in the occupation of $(1h_{11/2})_\nu$ is $<Q_0^2>$ moment increasing effect, whereas the increase in $(2d_{5/2})_\nu$ occupation is $<Q_0^2>$ moment decreasing effect. The results of their simultaneous increase keep the $<Q_0^2>$ moment nearly the same in $^{110-116}$Te. The observed systematics of 2^+ state in

Table 2. The sub-shell occupation numbers (protons) in the nuclei $^{110-116}$Te with

(i) VQP interaction

Te nuclei (A)	Sub-shell occupation number $3s_{1/2}$	$2d_{3/2}$	$2d_{5/2}$	$2f_{7/2}$	$1g_{7/2}$	$1h_{9/2}$	$1h_{11/2}$	$1i_{13/2}$
110	0.54	0.41	0.96	0.00	0.08	0.00	0.00	0.00
112	0.54	0.44	0.91	0.00	0.09	0.00	0.00	0.00
114	0.54	0.48	0.85	0.00	0.11	0.00	0.00	0.00
116	0.54	0.52	0.78	0.00	0.13	0.00	0.00	0.00
(ii) VQPOH interaction								
110	0.42	0.54	0.78	0.00	0.22	0.00	0.00	0.00
112	0.42	0.56	0.74	0.00	0.26	0.00	0.00	0.00
114	0.43	0.56	0.75	0.00	0.24	0.00	0.00	0.00
116	0.44	0.56	0.75	0.00	0.22	0.00	0.00	0.00

Table 3. The sub-shell occupation numbers (neutrons) in the nuclei $^{110-116}$Te with

(i) VQP interaction

Te nuclei (A)	Sub-shell occupation number $3s_{1/2}$	$2d_{3/2}$	$2d_{5/2}$	$2f_{7/2}$	$1g_{7/2}$	$1h_{9/2}$	$1h_{11}$	$1i_{13}$
110	0.66	1.69	4.02	0.05	1.23	0.01	0.26	0.03
112	0.81	1.83	4.36	0.16	1.91	0.03	0.77	0.09
114	0.91	1.90	4.51	0.38	2.48	0.04	1.68	0.07
116	0.97	1.92	4.60	0.70	2.98	0.05	2.80	0.00
(ii) VQPOH interaction								
110	0.42	1.23	3.11	0.51	1.22	0.01	1.47	0.00
112	0.44	1.23	3.03	0.92	1.38	0.07	2.89	0.01
114	0.59	1.40	4.04	0.88	1.93	0.09	2.97	0.08
116	0.84	1.61	4.78	0.89	2.59	0.10	3.07	0.09

$^{110-116}$Te indirectly imply that these isotopes should have nearly same $<Q_0^2>$ moment. The results of occupation numbers obtained in VQP interaction indicate that $(1h_{11/2})_\nu$ occupation increases from 0.26 in ^{110}Te to 2.80 in ^{116}Te (a quadrupole moment increasing effect) meaning thereby that 2^+ state should be lowered in energy value as we move from ^{110}Te to ^{116}Te which is not observed experimentally.

4. Yrast spectra

A projection calculation for the energy spectra of $^{110-116}$Te was carried out by employing the VQP and VQPOH models of two-body interaction. In Figure 1, the Yrast spectra for $^{110-116}$Te isotopes is displayed. In Figure 1, we have compared the experimental values of Yrast states with the theoretical values (Th.1 and Th.2). The spectra corresponding to Th.1 has been obtained by using VQP two-body interaction, whereas the spectra corresponding to Th.2 has been obtained by including a higher order octupole-octupole-plus-hexadecapole-hexadecapole interaction energy term in the two-body residual interaction (VQPOH). It turns out from our calculations that calculated spectra corresponding to Th.2. reproduces the observed Yrast spectra for $^{110-116}$Te reasonably well as compared to the Yrast spectra corresponding to Th.1. This level of agreement can be considered to be satisfactory because of a number of considerations. First, the calculation of Yrast spectra is a complex many body calculation involving a minimum of 10 valence particles for ^{110}Te and a maximum of 16 valence particles for ^{116}Te. Another noticeable fact is that the calculations are carried out for the entire set of the $^{110-116}$Te isotopes, with a single set of input parameters. Thus from the Figure 1, it is clear that the inclusion of the hexadecapole-hexadecapole interaction improves the agreement of the calculated spectra with the experiments. We have checked that there is very little difference occurring in the Yrast spectra when octupole-octupole interaction term is added to VQP interaction. It was observed by us that the inclusion of only octupole-octupole component of two-body residual interaction did not make any noticeable difference to the calculated nuclear structure quantities.

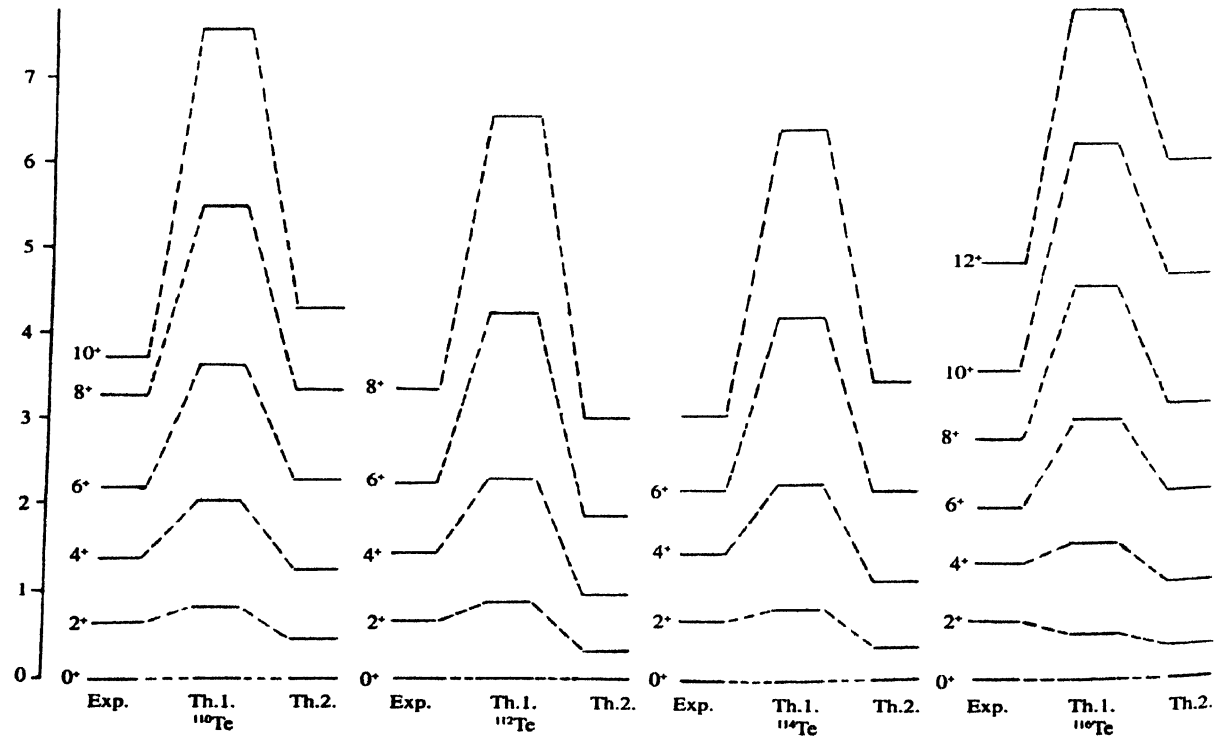

Figure 1. Comparison of the observed (Exp.) as well as the theoretical (Th.1 and Th.2) low-lying Yrast spectra in $^{110-116}$Te isotopes. Th.1 and Th.2 are the calculated spectra corresponding to the VQP and VQPOH models of the two-body interaction, respectively.

5. Transition probabilities B ($E2$) and quadrupole deformations (β_2)

The reliability and goodness of the HB wave-function is examined by calculating the B ($E2$; $0_1^+ \rightarrow 2_1^+$) values. Bhatt *et al* [25] have developed a formula for the calculation of B ($E2$; $0_1^+ \rightarrow 2_1^+$) transition probabilities from the values of intrinsic quadrupole moments of protons and neutrons, in the mass region of rare earth actinide nuclei. It has been justified by them that the B ($E2$; $0_1^+ \rightarrow 2_1^+$) in units of e^2b^2 are given by

$$B\ (E2;\ 0_1^+ \rightarrow 2_1^+) = (1.02 \times 10^{-5})A^{2/3}c^2_{\text{model}} \times [e_\pi <Q_0^2>_\pi + e_\nu <Q_0^2>_\nu]^2, \quad (4)$$

where $<Q_0^2>_\pi$ ($<Q_0^2>_\nu$) are the intrinsic quadrupole moments of valence protons (neutrons) and e_π and e_ν are the effective charges of the protons and neutrons, respectively.

Effective charges of the protons and neutrons are given by

$$e_\pi = (1+Z/A)e \text{ and } e_\nu = \varepsilon(Z/A)e.$$

They have recommended the use of $\varepsilon = 2.1$ and $c_{\text{model}} = (0.8 \pm 0.07)$ in the rare earth and actinide regions. We have used this formula for the calculation of the B ($E2$) values for the mass chain of $^{110-116}$Te isotopes.

In Table 4, we present a comparison of the calculated B ($E2$) values obtained with VQP and VQPOH interactions with the IBM and IBM2 values [11] for the $0_1^+ \rightarrow 2_1^+$ transitions in case of $^{110-116}$Te. It is satisfactory to note that the calculated B ($E2$) values are in satisfactory agreement with the values [11] for the $0_1^+ \rightarrow 2_1^+$ transitions in case of $^{110-116}$Te obtained with VQPOH interaction. In the same table, the calculated values for deformation parameter (β_2) alongwith the predicted value [13] have been presented. The deformation parameter β_2 is related to B ($E2$)$\uparrow$ by the formula suggested by Raman *et al.* [26] as

$$\beta_2 = (4\pi/3ZR_0^2)[B(E2)\uparrow/e^2]^{1/2}, \quad (5)$$

where R_0 is usually taken to be $1.2\ A^{1/3}$ fm and B ($E2$)$\uparrow$ is in units of e^2b^2.

From the systematics of the calculated β_2 values, it is noted that the set of values obtained with VQPOH interaction are in satisfactory agreement with the observed values.

Table 4. Comparison of the calculated B ($E2$: $0_1^+ \rightarrow 2_1^+$) values with the predicted IBM and IBM2 values of Ref. [11] and calculated β_2 values with the values of Ref. [13] in $^{110-116}$Te isotopes. The B ($E2$) values are in units of e^2b^2.

Te nuclei (A)	B ($E2$; $0^+ \rightarrow 2^+$)				β_2		
	Interaction				Interaction		
	VQP	VQPOH	IBM	IBM2	VQP	VQPOH	Ref. [13]
110	0.25	0.61	–	–	0.12	0.18	0.153
112	0.30	0.60	0.58	0.61	0.13	0.18	–
114	0.38	0.60	0.69	0.70	0.14	0.17	–
116	0.50	0.59	0.81	0.82	0.16	0.17	–

6. Conclusions

From the results of our calculations, the following conclusions can be drawn :

(i) The VAP calculations performed with VQPOH interaction reproduce correctly the observed deformation systematics in $^{110-116}$Te isotopes. The deformation systematics are seen to depend sensitively on the occupation probabilities of $(d_{5/2})_\nu$ and $(h_{11/2})_\nu$ orbits.

(ii) The results of VAP calculations show a marked improvement in the agreement with experiments when hexadecapole interactions are included in the two-body interaction. It seems that the two-body effective residual interaction in Te isotopes apart from having a dominantly quadrupole character, has a finite but small hexadecapole contribution which results in the generation of more accurate intrinsic wave function that simulates the observed properties of Te isotopes better than when only VQP interaction is employed.

(iii) The values of hexadecapole interaction parameters employed by us are the appropriate one's in this mass region, as with them, the HB wave function yields values of B ($E2$) and β_2 in satisfactory agreement with the experiments.

(iv) The quadrupole-quadrupole-plus-pairing model of two-body interaction with the inclusion of hexadecapole interaction gives a better overall understanding of properties of $^{110-116}$Te isotopes.

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